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Analytical investigation of porous pin fins with variable section in fully-wet conditions

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ABSTRACT

The present work investigates the temperature distribution, heat transfer rate, efficiency and optimization of porous pin fins in fully wet conditions. The thickness varies along the length of the fin and the lateral surface equation is defined as functions that include diversification fins (rectangular, triangular, convex parabolic and concave parabolic sections). Fins are made of aluminium and the tips of fins are insulated. Furthermore, it is assumed that the heat transfer coefficient depends on temperature and in the fin it changes according to temperature changes. In order to derivethe heat transfer equation, energy balance and Darcy model are used. After presenting the governing equation to obtain the temperature distribution, least squares method (LSM) is applied. Comparison of the results between analytical solution and numerical outcome (fourth order Runge–Kutta method) shows that LSM is a convenient and powerful method in engineering problems. Then the effects of various geometric and thermophysical parameters (power index for geometry (n), porosity, Biot number and relative humidity) on the dimensionless temperature fin, efficiency and heat transfer rate are examined. Optimum design analysis was also carried out.

1. Introduction

Use of porous surfaces and heat transfer in porous surfaces is prevalent in engineering science and industrial problems. Because of their effects on the heat transfer rate, many studies have been conducted on porous fins nowadays. Kiwan and Al-Nimr introduced the concept of using fins made up of porous materials by introducing the Darcy model for first time [1,2]. Hamdan and Al-Nimr [3] studied the increase in heat transfer between two parallel porous channels by using porous fins. Kundu and his colleagues presented an analytical study to determine the performance of convective porous fins with various geometries [4]. Numerical study on the heat transfer of cylindrical porous fins was carried out by Saedodin and Sadeghi [5] and they also achieved results similar to the previous studies.

In many industrial applications the heat transfer of fin is combined with cooling and dehumidification of ambient air. Assuming that fin surface temperatures is below the dew point temperature of the surrounding air, heat and mass transfer occurs simultaneously in fin surface because of condensation of air humidity. Naphon studied the annular fin under dry-surface, partially wet-surface, and fully wet-surface conditions [6]. In many , a linear function is used to determine the

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relationship between the dry-bulb temperature and specific humidity [7–10] to analyse the performance of the wet fins. Assuming the temperature of the tip is dew point temperature [11], Sharqawy and Zubair determined the fin efficiency of straight wet fins by considering a new linear relationship between specific humidity and sensible temperature. Sabbaghi and his colleagues [12] investigated the efficiency of a semi-spherical fin. The results demonstrate that the overall efficiency of the fin depends on the fin surface conditions and the relative humidity does not have a large effect on efficiency.

The best extended surfaces (fins) are the fins that provide the maximum heat transfer, or the maximum temperature difference. Then the best situation should be found in which all the necessary conditions can be satisfied simultaneously. Several studies have already been done in this area. Yu and Chen studied on optimization of circular fins with variable thermal parameters [13]. For electronics cooling applications, Shuja presented an optimized geometry of fins, based on analytical methods [14]. He also investigated the effects of Reynolds number on output data. Analytical analysis of performance and optimization of circular and SRC profiles fin is conducted by Kundu [7,9]. He concluded that the heat transfer rate increases with the increase of ambient temperature. Kundu and Bhanja examined the influence of some independent parameter on efficiency and optimization of porous fin [15]. The results indicate that the optimum condition of porous fins is a function of ambient and base temperatures. Analytical solution for heat transfer equations of fins with variable geometries under wet conditions is determined by Kundu and Lee[16]. Bouaziz and Aziz introduced the least squares method (LSM) to solve the equations of fin [17,18]. They also illustrate that LSM is a simple and accurate method in comparison with other analytical methods. Hatami, Hasanpour and Ganji [19] have used three analytical methods for analysing the performance of longitudinal fin with temperature-dependent heat generation and indicate that the least squares method is more accurate in comparison with other methods. Analysis of straight fins with temperature-dependent thermal conductivity was performed by Joneidi, Ganji, and Babaelahi [20] using differential transformation method. Recently, Hatami and Ganji studied the thermal behaviour of porous fins with different profiles and different materials in terms of convection and radiation. The results illustrate that LSM is a powerful, efficient method and it also reduces the size of calculations [21,22]. They also studied the heat transfer in porous wet circular fins and they used least squares method to predict the temperature distribution.

Even though use of pin fins in some kinds of heat exchangers (Air Conditioning, Aeronautics Industry) are not prevalent, but this kind of extended surfaces are widely used in the electronics industry. So the effects of pin fins with variable profiles in refrigeration and providing an optimal design is investigated for the first time in the present study.

2. Mathematical formulation

The pin porous fins with variable geometries are presented in Fig. 1, under fully wet conditions. The assumptions are as follows: 1) fins are in steady state; 2) moist air flows with constant and uniform velocity around the fins; 3) fin surface is porous, homogeneous and isotropic; 4) there is no any heat generation and contact resistance at the base of the fin; 5) temperature of the fin surface is between the dew point and the fin base temperature. According to the two-dimensional schematic of fin profile, which is shown in Fig. 1e and the aforementioned assumptions, the governing equation can be

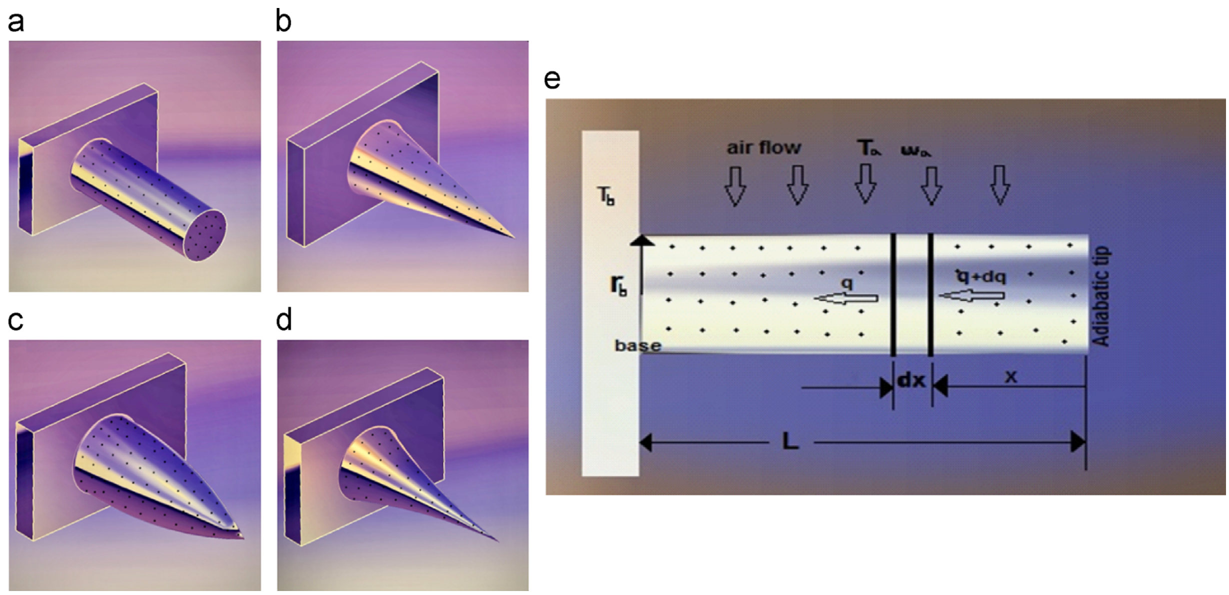

Fig. 1. Schematics of fully wet porous pin fins with variable section (a) Rectangular, (b) Triangular, (c) Convex parabolic and (d) Concave parabolic section, (e) Two-dimensional schematic of rectangular profile.

written as

$$q_x - q_{x+dx} = hpd_x(1-\varphi)(T-T_a) + \dot{m}C_p(T-T_a) + h_d i_{fg} pd_x(1-\varphi)(\omega-\omega_a) \tag{1}$$

where φ is the porosity. The mass flow rate of the fluid passing through porous media is [1,19]:

$$\dot{m} = \rho v A = \rho v(2\pi r dx) \tag{2}$$

Considering the Darcy model, the fluid velocity passing through the fins can be calculated using [11]

$$V = \frac{gK\beta(T-T_a)}{\nu} \tag{3}$$

Using the Chiltone–Colburn analogy [11], the following relationship exists between heat and mass transfer

$$\begin{cases} \frac{h}{h_d} = C_p \times Le^{2/3} \\ B = \frac{i_{fg}}{C_p \times Le^{2/3}} \end{cases} \rightarrow \quad h_D \times i_{fg} = h \times B \tag{4}$$

As mentioned before, the specific humidity is approximated by Sharqawy and Zubair [11] as a linear function of dry temperature in which the fin surface temperature is between dew point and the base temperature:

$$\omega = a_1 + b_1 T \tag{5}$$

$$a_1 = \omega_b - \frac{\omega_{dp} - \omega_b}{T_{dp} - T_b} T_b, \mathrm{b}_1 = \frac{\omega_{dp} - \omega_b}{T_{dp} - T_b} \tag{6}$$

Considering Fourier's law of conduction and substituting the above equation into Eq. (1), we have

$$-\frac{d}{dx}\left(-k_{eff} A \frac{dT}{dx}\right) = hPdx(1-\varphi)(T-T_a) + \frac{\rho g K\beta(T-T_a)}{\nu} v(2\pi r dx) + hPdx \times B(1-\varphi)(a_2 + b_2 T_a - \omega_a) \tag{7}$$

The effective conductivity, temperature-dependent heat transfer coefficient and fin area, respectively, are defined as follows:

$$k_{eff} = \varphi k_f + (1-\varphi)k_s \tag{8}$$

$$h = h_a \left[\frac{T-T_a}{T_b - T_a}\right]^p \tag{9}$$

$$A = \pi r^2 \tag{10}$$

In the present study, various profiles of pin fin are investigated and the equation of lateral surface is defined by the following function:

$$r = r_b \left(\frac{x}{L}\right)^n = r_b X^n \begin{cases} n = 0 \text{ Rectangular profile} \\ n = 1 \text{ Triangular profile} \\ n = \frac{1}{2} \text{ Convex parabolic profile} \\ n = 2 \text{ Cocave parabolic profile} \end{cases} \tag{11}$$

In order to simplify the energy equation, the dimensionless parameters are defined as follows:

$$(X;\psi;\theta) = \left(\frac{x}{L};\frac{r_b}{L};\frac{T}{T_b}\right) \tag{12}$$

$$(Bi;R_1;\xi) = \left(\frac{h_a r_b}{k_s};\frac{2\rho C_p g K\beta T_b}{\nu h_a};\frac{k_{eff}}{k_s}\right) \tag{13}$$

By substituting the dimensionless parameters into Eqs. (12) and (13), we obtain the following equation:

$$X^n \frac{d^2\theta}{dX^2} + 2nX^{n-1}\frac{d\theta}{dX} = m_1(\theta-\theta_a)^2 + \frac{Bm_0{}^2 m_2}{(1-\theta_a)^P}(\theta-\theta_a)^P + \frac{m_3{}^2}{(1-\theta_a)^P}(\theta-\theta_a)^{P+1} \tag{14}$$

where the physical parameters m_0, m_1, m_2 and m_3 are defined to summarize the energy equation,

$$m_0{}^2 = \frac{2h_a(1-\varphi)L^2}{k_{\mathrm{eff}} r_b} = \frac{2Bi(1-\varphi)}{\xi\psi^2} \tag{15}$$

$$m_1 = \frac{Bi \cdot R_1}{\xi\psi^2} \tag{16}$$

$$m_2 = \frac{a_2 + b_2 T_a - \omega_a}{T_b} \tag{17}$$

$$m_3{}^2 = m_0{}^2(1+Bb_2) \tag{18}$$

So the energy equation for specified profiles (rectangular, triangular, convex parabolic and concave parabolic profiles) in the current study are as follows respectively:

$$\frac{d^2\theta}{dX^2} = m_1(\theta-\theta_a)^2 + \frac{Bm_0{}^2m_2}{(1-\theta_a)^P}(\theta-\theta_a)^P + \frac{m_3{}^2}{(1-\theta_a)^P}(\theta-\theta_a)^{P+1} \tag{19}$$

$$X\frac{d^2\theta}{dX^2} + 2\frac{d\theta}{dX} = m_1(\theta-\theta_a)^2 + \frac{Bm_0{}^2m_2}{(1-\theta_a)^P}(\theta-\theta_a)^P + \frac{m_3{}^2}{(1-\theta_a)^P}(\theta-\theta_a)^{P+1} \tag{20}$$

$$X^{1/2}\frac{d^2\theta}{dX^2} + X^{-(1/2)}\frac{d\theta}{dX} = m_1(\theta-\theta_a)^2 + \frac{Bm_0{}^2m_2}{(1-\theta_a)^P}(\theta-\theta_a)^P + \frac{m_3{}^2}{(1-\theta_a)^P}(\theta-\theta_a)^{P+1} \tag{21}$$

$$X^2\frac{d^2\theta}{dX^2} + 4X\frac{d\theta}{dX} = m_1(\theta-\theta_a)^2 + \frac{Bm_0{}^2m_2}{(1-\theta_a)^P}(\theta-\theta_a)^P + \frac{m_3{}^2}{(1-\theta_a)^P}(\theta-\theta_a)^{P+1} \tag{22}$$

Considering the assumption that the fin tip is insulated, we have the following boundary conditions:

$$\theta(1) = 1, \qquad \frac{d\theta}{dX}\Big|_{X=0} = 0 \tag{23}$$

3. Describe least square method (LSM) and applied to the problem

3.1. Describe least square method

To understand the least squares method in the present study, Hatami and Ganji's [22] definition is exactly mentioned here. The least squares method is one of the weighted residual methods which is constructed on minimizing the residuals of the trial function introduced to the nonlinear differential equation. To understand the principle of LSM, consider a differential operator D to act on a function u to produce a function p:

$$D(u(x)) = p(x) \tag{24}$$

It is considered that u is estimated by a function, $\tilde{u}$, c which is a linear combination of fundamental functions chosen from a linearly independent set:

$$u \cong \tilde{u} = \sum_{i=1}^{n} c_i\varphi_i \tag{25}$$

By substituting Eq. (25) into the differential operator, D, the result of the operations generally is not $p(x)$ and a difference will appear. Hence an error or residual will exist as follows:

$$R(x) = D(\tilde{u}(x)) - p(x) \neq 0 \tag{26}$$

The main concept of LSM is to force the residual to zero in some average sense over the domain. So,

$$\int_x R(x)\, W_i(x) = 0 \quad i = 1, 2, \dots, n \tag{27}$$

where the number of weight functions, W_i, is accurately equal to the number of unknown coefficients c_i in $\tilde{u}$. The result is a set of n algebraic equations for the undefined coefficients c_i. If the continuous summation of all the squared residuals is minimized, the rationale behind the LSM's name can be seen. In other words, a minimum of

$$S = \int_x R(x)R(x)dx = \int_x R^2(x)dx \tag{28}$$

In order to achieve a minimum of this function, Eq. (28), the derivatives of S with respect to each of the unknown parameters should be zero, i.e.

$$\frac{\partial S}{\partial c_i} = 2\int_x R(x)\frac{\partial R}{\partial c_i}dx = 0 \tag{29}$$

Comparing with Eq. (29), the weighted functions for LSM will be

$$W_i = 2\frac{\partial R}{\partial c_i} \tag{30}$$

Because the coefficient "2" coefficient can be eliminated, it is can be negligible in the equation. So the weighted functions, W_i, for the least squares method are the derivatives of the residuals with respect to the unknown constants:

$$W_i = \frac{\partial R}{\partial c_i} \tag{31}$$

3.2. The LSM applied to the problem

Considering that the boundary conditions must satisfy in trial solution, the trial solution can be written as

$$\theta(X) = 1 + c_1(1-X^2) + c_2(1-X^3) + c_3(1-X^4) \tag{32}$$

After obtaining the residual function by substituting Eq. (32) into Eq. (7), three equations with three unknown coefficients (c_1, c_2, c_3) will be obtained. These coefficients will be calculated by solving this system of equations. Then by substituting c_1, c_2, c_3 into the trial solution the temperature distribution for different geometries of fully wet porous pin fins will be determined. As an example, the following equations demonstrate temperature distribution when $\psi = 0.05, Bi = 0.001$ φ=0.2, h=25 (W m^{-2} K^{-1}), k_f=0.0262 (W K^{-1} m^{-1}), p=Le=1, RH=60%, T_b=30 °C and T_a=50 °C.

I. **Rectangular profile (*n*=0):**

$$\theta(X) = 1.286895330 - 0.2475994880X^2 + 0.1194640816X^3 - 0.1587599237X^4 \tag{33}$$

II. **Triangular profile (n=1):**

$$\theta(X) = 1.296762142 - 0.5645930376X^2 + 0.4854006756X^3 - 0.2175697802X^4 \tag{34}$$

III. **Convex parabolic profile (n=1/2):**

$$\theta(X) = 1.556099191 - 0.6132860258X^2 + 0.3662894570X^3 - 0.3091026224X^4 \tag{35}$$

IV. **Concave parabolic profile (n=2):**

$$\theta(x) = 1.344782274 - 0.9570213304X^2 + 0.9555909514X^3 - 0.3433518947X^4 \tag{36}$$

Table 1 presents the thermophysical properties of aluminium which is used as a base material in porous fin. CATT software (Computer Aided Thermodynamics Tables) is used for calculating constants a_1 and b_1 in Eq. (6), which vary with variations in relative humidity. This can be seen in Table 2.

3.3. Calculation of heat transfer rate and efficiency

By applying the Fourier's law of heat conduction at the base of the fin, the dimensionless form of heat transfer rate per unit area in actual condition can be written as [4,16]

$$Q = \frac{q}{(k_s{}^2\pi T_b/h_a)} = \xi\psi Bi\frac{d\theta}{dX}\bigg|_{X=1} \tag{37}$$

The maximum heat transfer rate when the entire fin surface is at the base temperature (ideal condition) can be calculated as [4,16]

$$Q_{\text{ideal}} = \frac{q_{\text{ideal}}}{(k_s{}^2\pi T_b/h_a)}$$

Table 1
Thermo physical properties of fluid and porous material.

ρ(kg.m^{-3})(Al)	K_f(W.K^{-1}.m^{-1})(air)	K_s(W.K^{-1}.m^{-1})(Al)	C_p(kJ.kg^{-1}.K^{-1})(Al)
2700	0.0262	237	1.006

Table 2
Necessary data for calculating a_1 and b_1 constants obtained from CATT software.

RH (%)	T_a(°c)	ω_a	T_{dp}(°c)	ω_{dp}	T_b(°c)	ω_b
60	50	0.04906	30.74	0.02843	30	0.272
80	50	.06718	45.57	0.06717	30	0.0272
100	50	0.08631	50	0.08631	30	0.0272

$$Q_{\text{ideal}} = \xi\psi Bi\left(\left(m_3{}^2(1-\theta_a)+m_1(1-\theta_a)^2+m_0{}^2m_2B\right)\times\int_0^1\left(X^n\sqrt{1+\left(\frac{n.r_bX^{n-1}}{L}\right)^2}\right)dX\right) \tag{38}$$

The ratio of the actual heat transfer to ideal heat transfer generally indicates the fin efficiency [4,16,22]:

$$\eta=\frac{Q_{\text{fin}}}{Q_{\text{ideal}}}=\frac{(d\theta/dX)|_{X=1}}{\left(m_3{}^2(1-\theta_a)+m_1(1-\theta_a)^2+m_0{}^2m_2B\right)\times\int_0^1\left(X^n\sqrt{1+(n.r_bX^{n-1}/L)^2}\right)dX} \tag{39}$$

3.4. Optimization

It is clear that the rate of heat transfer at fin decreases with increasein the fin length. Therefore, to achieve the accost-effective Fin, it is necessary to know the optimum length. Hence, researchers and scientists are continuously making efforts to determine the optimum shape in two ways that give the same result. The heat transfer rate for a specified fin volume maximizes or minimizes the fin volume for a given heat transfer rate. In this study optimization is performed in order to obtain the highest rate of heat transfer for a specified fin volume. The result of this optimization shows the necessity to use appropriate strategies to achieve a convenient fin [4,16]. The volume of the fin can be written in a dimensionless form as

$$U=\frac{h_a{}^3V}{k_s{}^3\pi}=\frac{Bi^3}{\psi(2n+1)} \tag{40}$$

where n is a constant coefficient for the respective profiles.

It is obvious that the fin volume is a function of Bi and ψ for all profiles. Heat transfer can be considered as a function of the single variable ψ or as a function of the two variables ψ and n via Eq. (40). Following the above description, optimal conditions can be calculated conveniently for the rectangular, triangular, convex and concave profiles as

$$\begin{pmatrix}\frac{\partial Q}{\partial\psi}\\ \frac{\partial Q}{\partial n}\end{pmatrix}=\begin{pmatrix}0\\0\end{pmatrix} \tag{41}$$

The solution of Eq. (41) yields the optimum ψ and n for a fin volume design constraint. The optimum Bi can be estimated from Eq. (40) after substituting the previously calculated optimum values. Finally, the maximum heat transfer rate can be obtained from Eq. (38).

4. Results and discussions

In the current study after obtaining the governing equation in fully wet porous pin fins with variable section (Fig. 1), the LSM has been applied to solve them. Then the LSM results are compared with the fourth order Runge-Kutta numerical method;Fig. 2a shows this comparison. The constants parameters in this figure are: $Bi=0.001$,$\psi=0.05$, $R_1=0.1$, $P=0$, $\varphi=0.2$, $RH=100\%$, $h_a=25$, $k_f=0.0262$, $T_b=50$, $T_a=30$. As seen for all geometries, LSM has an excellent agreement with numerical results and can be introduced as a simple and powerful analytical method for these kinds of problems. Fig. 2a also indicates the different temperature distributions for different geometries. It is concluded from this figure, that concave parabolic profile has the highest dimensionless temperature. The variation of fin efficiency with the variation of Biot number (Bi) is shown in Fig. 2b for rectangular, triangular, convex parabolic and concave parabolic profiles. For all the fin geometries, the fin efficiency decreases with increase in Bi. On comparison of four profiles, it is found that the fin efficiency of concave parabolic profile is the maximum. As mentioned before, the ratio of the actual heat transfer to ideal heat transfer generally indicates the fin efficiency (Eq. (39)). Therefore, the heat transfer rate must be calculated. Some authors, e.g. Kundu[14], used base temperature gradient to present the q_{fin} as used in this study. In Fig. 2b it is clear that fins with higher base gradient temperature have higher efficiency. Fig. 2c indicates the sample result obtained from optimization study for the aforementioned profiles. The dimensionless fin volume is taken at $U=10^{-6}$ m^3. For each profile, in the beginning, the rate of heat transfer increases to a maximum value and then starts decreasing. The turning point of the curves indicates an optimum design condition for a given fin volume. The results show that the pin porous fins with concave parabolic profile have slightly higher heat transfer rate in comparison with triangular profile and the magnitude of heat transfer rate of

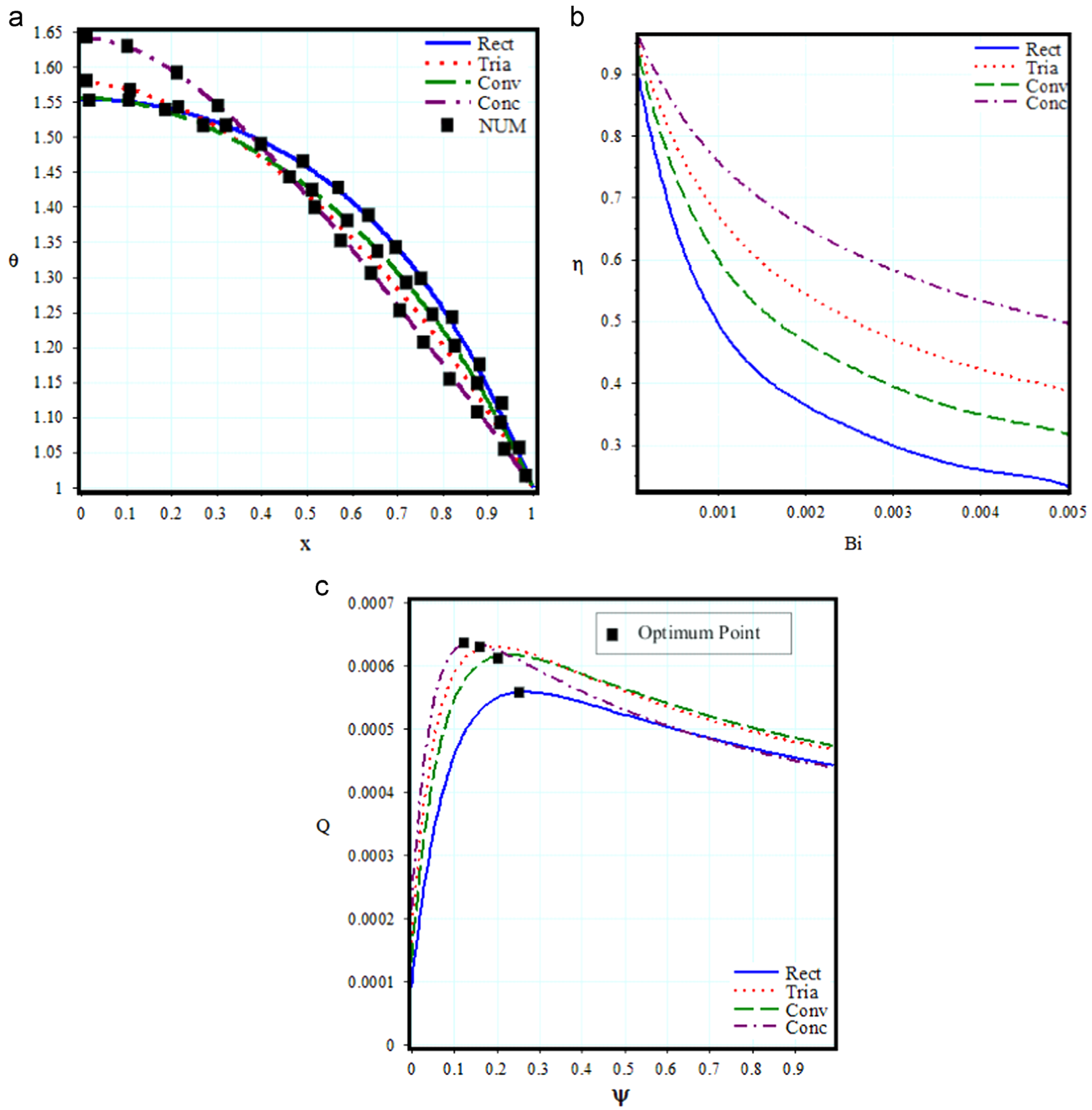

Fig. 2. (a) Comparison of LSM results with numerical solution for different section shapes, (b) Fin efficiency as a function of Bi obtained from various profiles when RH=60%, $\Psi=0.05$, $R_1=0.1$, $P=0$, $\varphi=0.2$, (c) Dimensionless heat transfer rate as a function of Ψ obtained from various profile for a design condition $U=10^{-6}$, $R_1=0.1$, $P=0$, $\varphi=0.2$, RH=60%.

rectangular profile is the lowest. However the rectangular profile is used in most engineering applications because of its ease in construction, but the results show that the concave parabolic profile with a low optimum ψ value is appropriate in comparison to other profiles. It is interesting to note that the optimum value of ψ decreases with increase in the power index of geometry (n).

The effect of relative humidity on the temperature distribution for various profiles is shown in Fig. 3a. It is evident that the that temperature distribution is increased by increasing the RH percentage. Therefore high relative humidity makes higher surface temperature due to release of latent heat of condensation and because the fins tip temperature is below the dew point of air. The influence of relative humidity on both efficiency and heat transfer rate is shown in Fig. 3b and c. As can be seen in Fig. 3b, for all four profiles, fin efficiency is reduced by increasing the *RH* percentage. The variation of dimensionless heat transfer rates with the variation of *RH* and ψ for all given profiles in the present study is plotted in Fig. 3c. It can be understood that for a given fin volume, the optimum design condition of a fin is strongly influenced by the

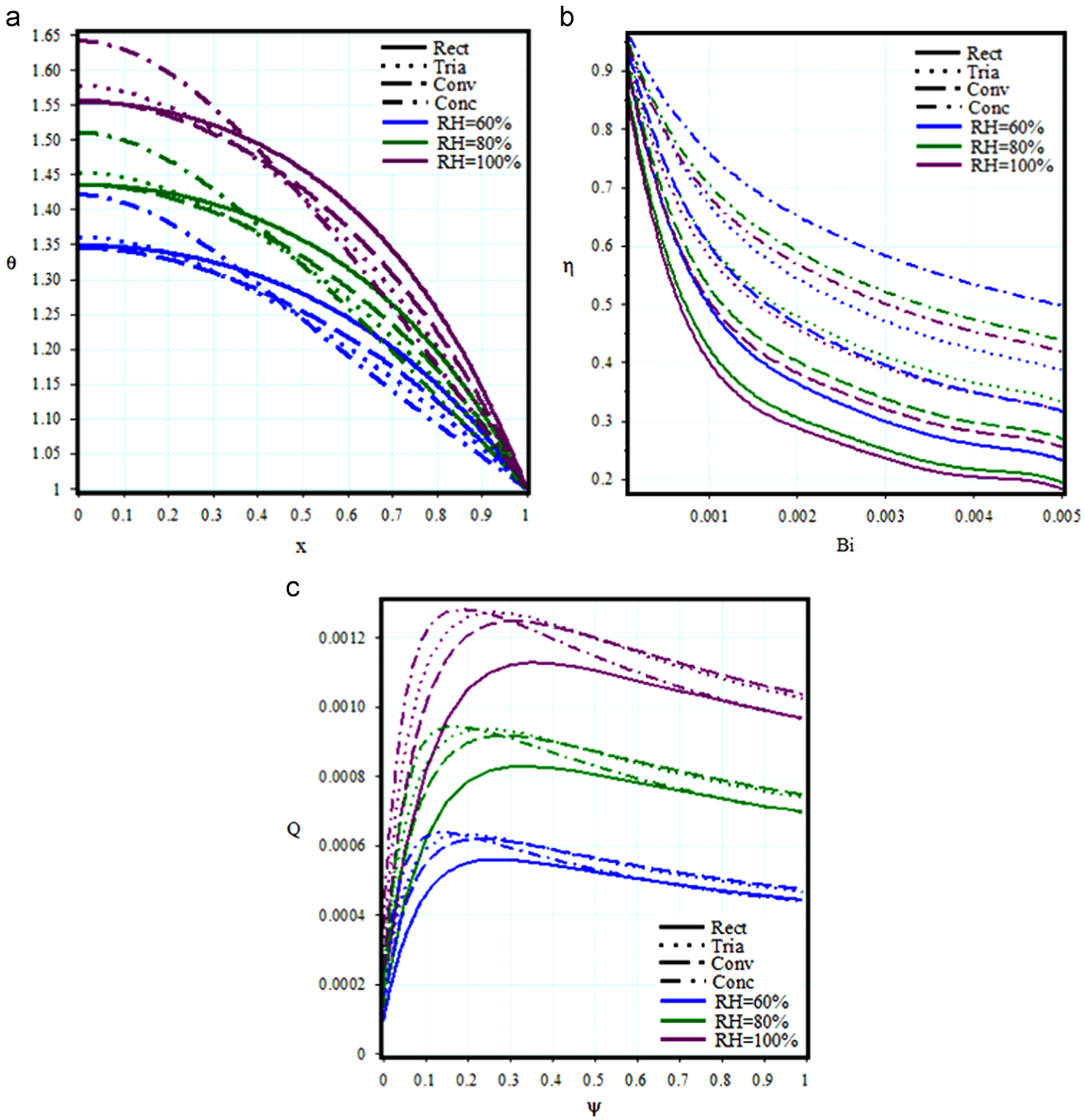

Fig. 3. Effect of relative humidity (RH) (a) temperature distribution, (b) Fin efficiency, (c) Heat transfer rate for various profile when $P=0$, $R_1=0.1$, $\varphi=0.2$.

relative humidity. This figure also shows that the dimensionless heat transfer rate is preferred for the concave parabolic profiles.

Fig. 4 shows the influence of porosity (φ) on temperature distribution, fin efficiency and heat transfer rate respectively for all of the specified profiles. According to Fig. 4a temperature distribution decreases with increase in the porosity parameter for all geometries. Actually due to the lack of solid material, the high porosity value reduces the effective thermal conductivity of the fin. At the same time, convective heat transfer increases due to more fluid passing through pores. This figure predicts that the variation of dimensionless temperature for the concave parabolic profile is higher than other profiles. Fig. 4b implies that the fin efficiency increases the function of porosity. The fin efficiency increases slowly with φ up to $\varphi=0.9$ for all profiles. In Fig. 4c, for four given profiles, it is clear that when the porosity increases, the dimensionless heat transfer rate decreases and the optimum ψ value decreases with increase in the porosity.

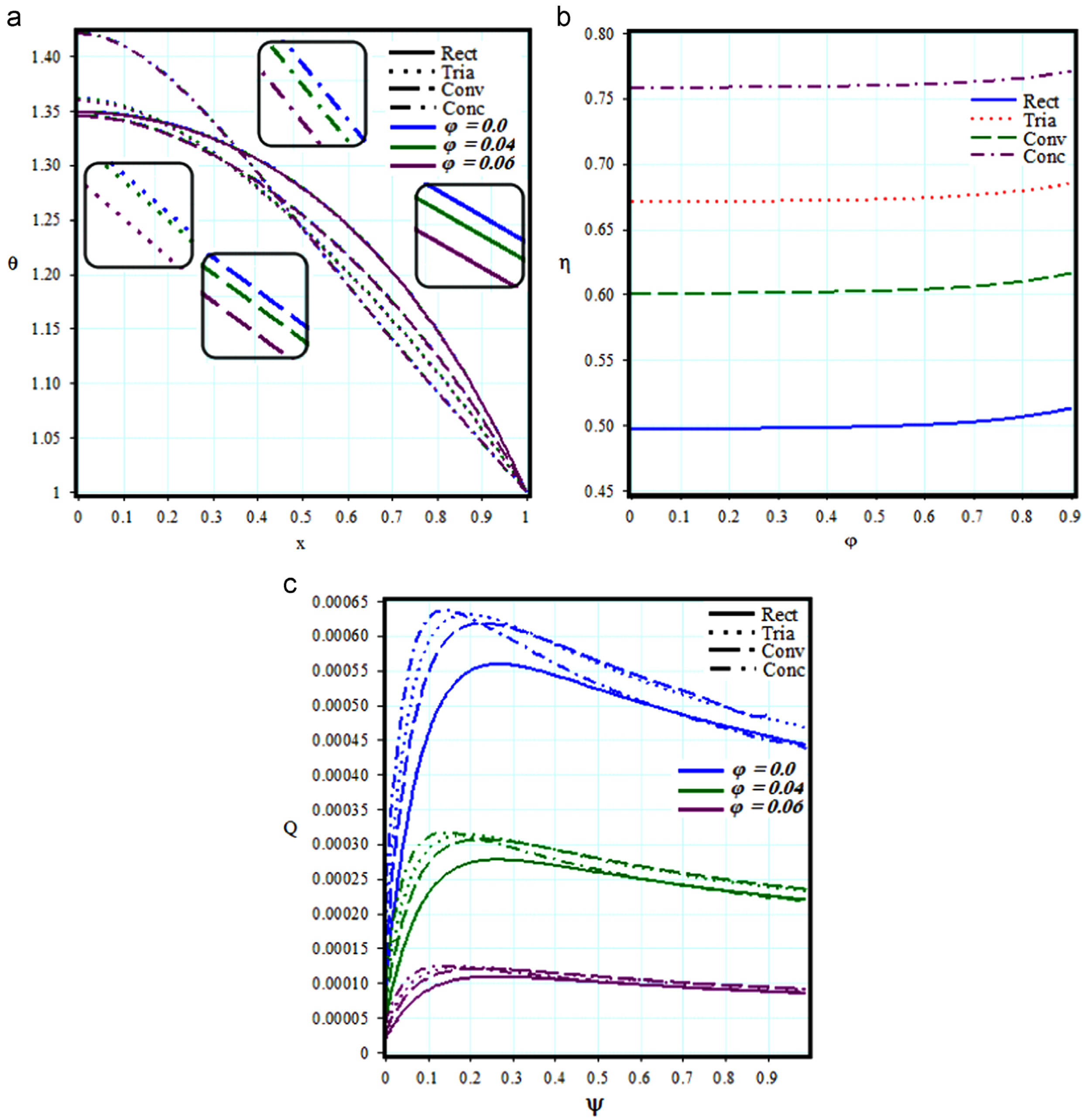

Fig. 4. Effect of porosity on (a) temperature distribution, (b) Fin efficiency, (c) Heat transfer rate, for various profile when RH=60%, $\Psi=0.05$, $R_1=0.1$, $P=0$.

The effects of power index of temperature-dependent h (P) on temperature distribution, fin efficiency and heat transfer rate for all of given profiles are displayed in Fig. 5 respectively. It is illustrated in Fig. 5a that the magnitude of temperature decreases with increasing P. This means that the fin surface temperature is related to the nature of the heat transfer coefficient. Fig. 5b reveals the when h is not dependent on the fin temperature ($P=0$), fin has higher efficiency and with improvement of P the fin efficiency decreases. This subject includes four specified profiles. Moreover, as expected, in this figure, in comparison with other profiles, the concave parabolic profile has highest efficiency. In Fig. 5c it is obvious that the dimensionless heat transfer rate is a decreasing function of P. When convection heat transfer coefficient is independent of temperature, the heat transfer rate has a maximum value and on decreasing the power index of temperature-dependent h, the heat transfer rate increases. It can be highlighted that the optimum Ψ value improved when P is increased.

Fig. 6a is plotted to illustrate the effects of fluid flow parameter R_1 on efficiency. Fin efficiency with variation of Biot number is decreasing due to increase in R_1. Near $Bi=0$, variation of R_1 does not have much influence on the fin efficiency but

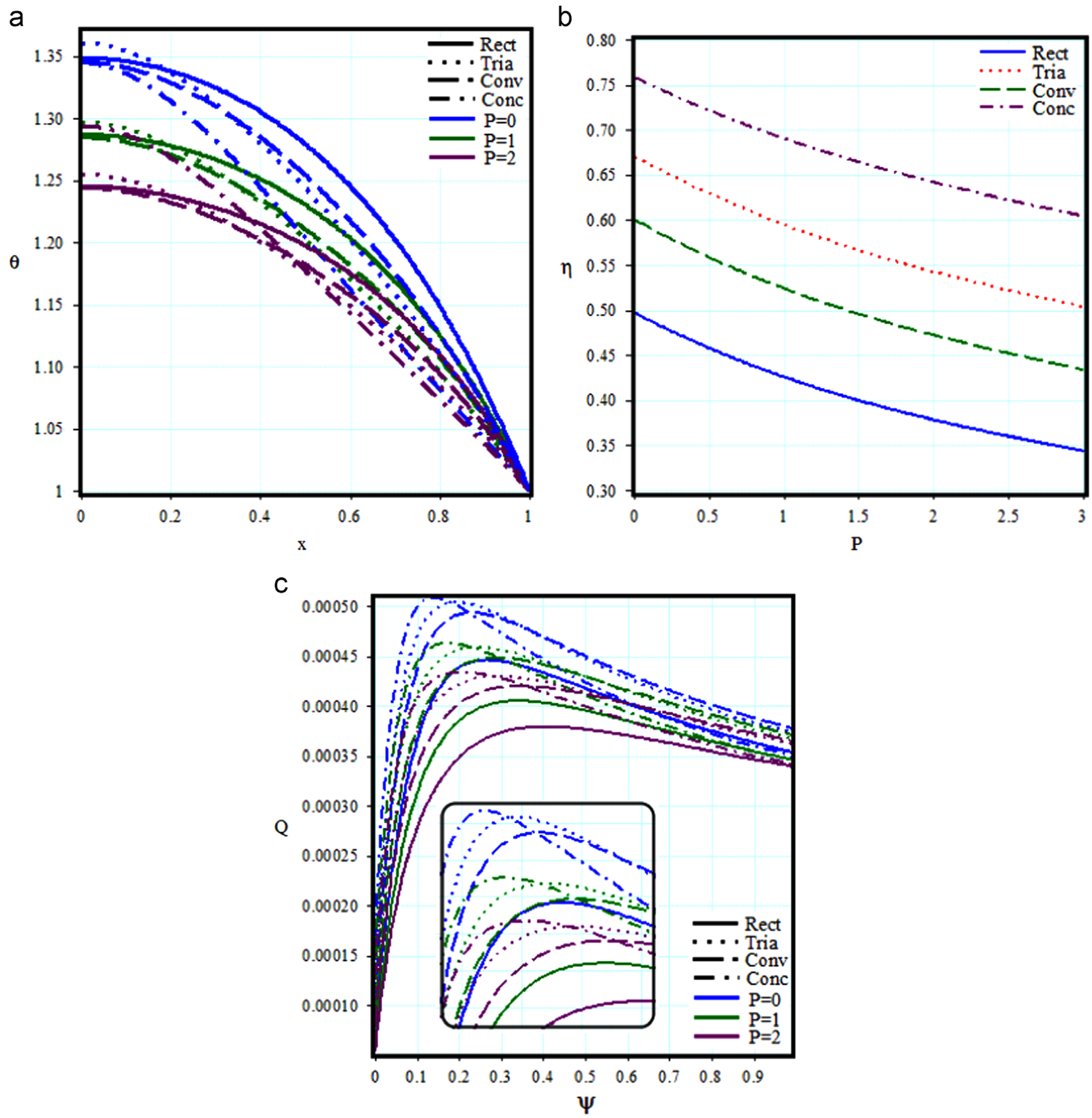

Fig. 5. Effect of convection power index on (a) temperature distribution, (b) Fin efficiency, (c) Heat transfer rate, for various profile when Bi = 0.001, $\Psi = 0.05$, $R_1 = 0.1$, $\varphi = 0.2$.

with increase in Bi the effect of R_1 increases. Fig. 6b indicates that the dimensionless heat transfer rate changes with the variation of ψ and R_1. The heat transfer rate decreases with the fluid flow parameter R_1 for the adopted profiles. The optimum ψ value improves if R_1 is decreased, so in this figure the maximum of ψ value belongs to the rectangular profile when $R_1 = 0.1$.

All previous results, in the present study, for pin fins are considered for four arbitrary constants of power index for geometry values ($n = 0$, 0.5, 1, 2). In order to obtain the maximum heat transfer in fully wet porous pin fins n can be determined from optimal conditions solution. Fig. 6c displays the influence of n on heat transfer rate for a design condition ($U = 10^{-6}$, $RH = 60\%$, $\psi = 0.05$, $R_1 = 0.1$, $\varphi = 0.2$). With increasing n from zero, the heat transfer rate increases to reach the maximum value; on further increasing n, the heat transfer rate decreases. There is an optimum point of n ($n = 6.36$), which is visible inFig. 6c.

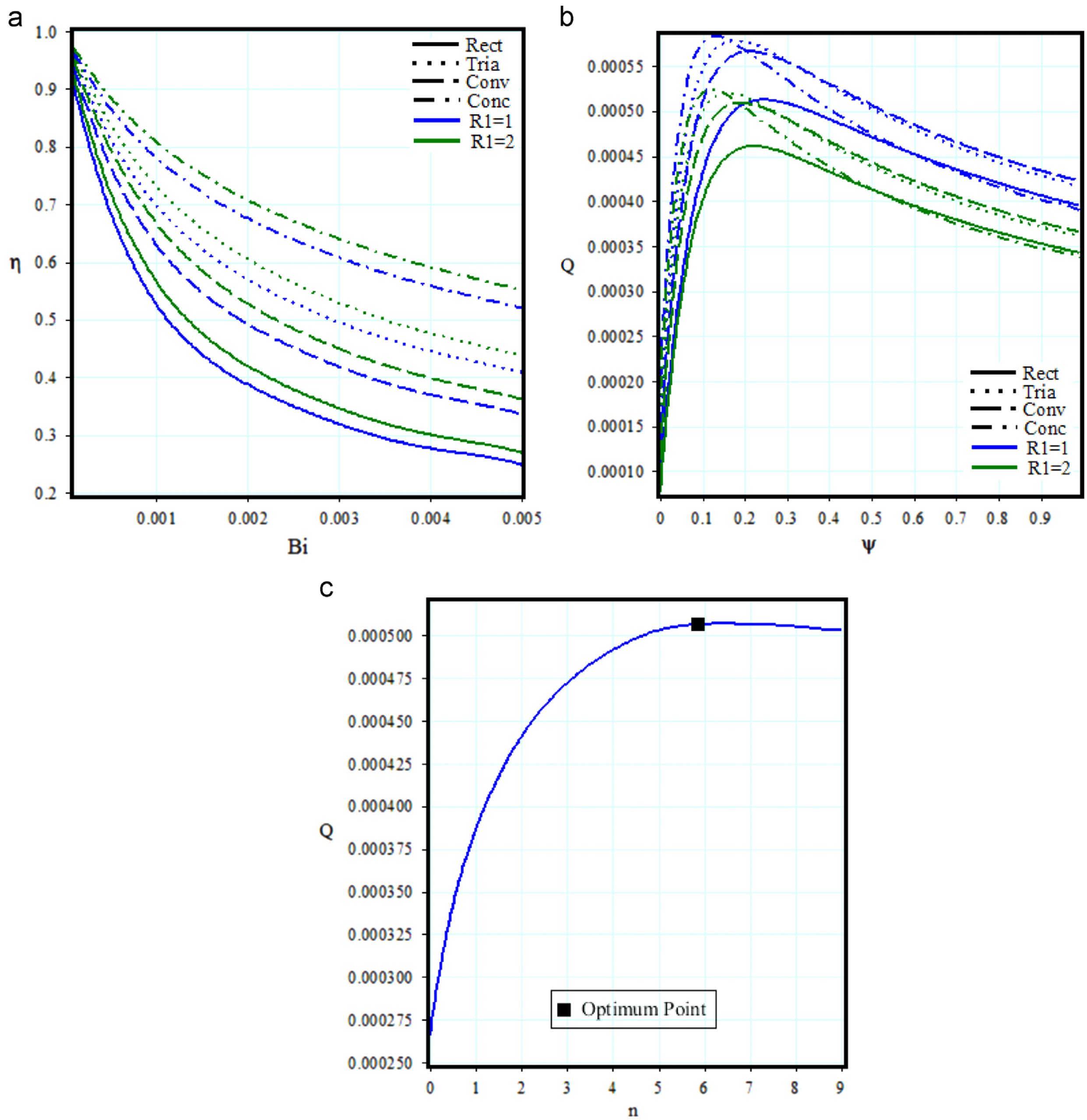

Fig. 6. Effect of fluid flow parameter (R_1) on **a)** Fin efficiency **b)** Heat transfer rate for various profile $P=0$, $\varphi=0.2$, RH=60%. **c)** Heat transfer rate of fully wet porous pin fins as a function of Power index for geometry (n) when $U=10^{-6}$, RH=60%, $\Psi=0.05$, $R_1=0.1$, $\varphi=0.2$.

5. Conclusion

In this paper, heat transfer equation, fin efficiency and heat transfer rate for porous pin fins in fully wet conditions with rectangular, triangular, convex parabolic and concave parabolic profiles are studied. The following important points can be concluded from the present study:

- Comparison of the results of the analytical solution with the numerical outcomes shows~~s~~ that LSM is a convenient and powerful method in engineering problems.

- Temperature distribution is increased by increasing the RH percentage. So high relative humidity makes higher surface temperature due to release of latent heat of condensation and because the fin's tip temperature is below the dew point of air.
- However the rectangular profile is used in most engineering applications because of its ease in construction, but the results show that the concave parabolic profile with a low optimum ψ value is appropriate in comparison to other profiles.

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